Multiple Equilibria, Chaos, and Aging Dynamics in Large Interacting Ecosystems

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Bunin, GB, Cammarota, New Journal of Physics 2018
Roy, Bunin, GB, Cammarota J Phys A 2019
.... a couple to appear

“Traditional” ecosystems

“Modern” ecosystems

MANY INTERACTING SPECIES

- Communities formed by individuals belonging to different species.
- Interactions between individuals intra and inter species.
- Competition for resources--Cooperation.
- Abundances of species vary dynamically due to the births and deaths.

Lotka-Volterra equations for ecosystems

$$\frac{dN_i}{dt} = N_i \left[r_i(K_i - N_i) - \sum_{j,(j \neq i)} \alpha_{ij} N_j \right] + \xcancel{\sqrt{N_i}\eta_i(t)} + \lambda_i$$

$$i = 1, \ldots, S$$

$N_i \geq 0$ abundance of species i

S is the number of species

Well-mixed population: no-space dependence

Large number of species
(S~50-100 is large)

Mean-Field Model of Ecosystems

$$\frac{dN_i}{dt} = N_i \left[r_i(K_i - N_i) - \sum_{j,(j \neq i)} \alpha_{ij} N_j \right] + \sqrt{N_i}\eta_i(t) + \lambda_i$$

Main assumption: complex -> random

(May in ecology & Wigner in physics)

(Determining interactions network: a key inference problem)

α_{ij} i.i.d. Random Variables

$$\langle \alpha_{ij} \rangle = \frac{\mu}{S} \qquad \langle \alpha_{ij}^2 \rangle_c = \frac{\sigma^2}{S}$$

$$\langle \alpha_{ij} \alpha_{ji} \rangle_c = \gamma \langle \alpha_{ij}^2 \rangle_c$$

$\gamma = 1$ symmetric

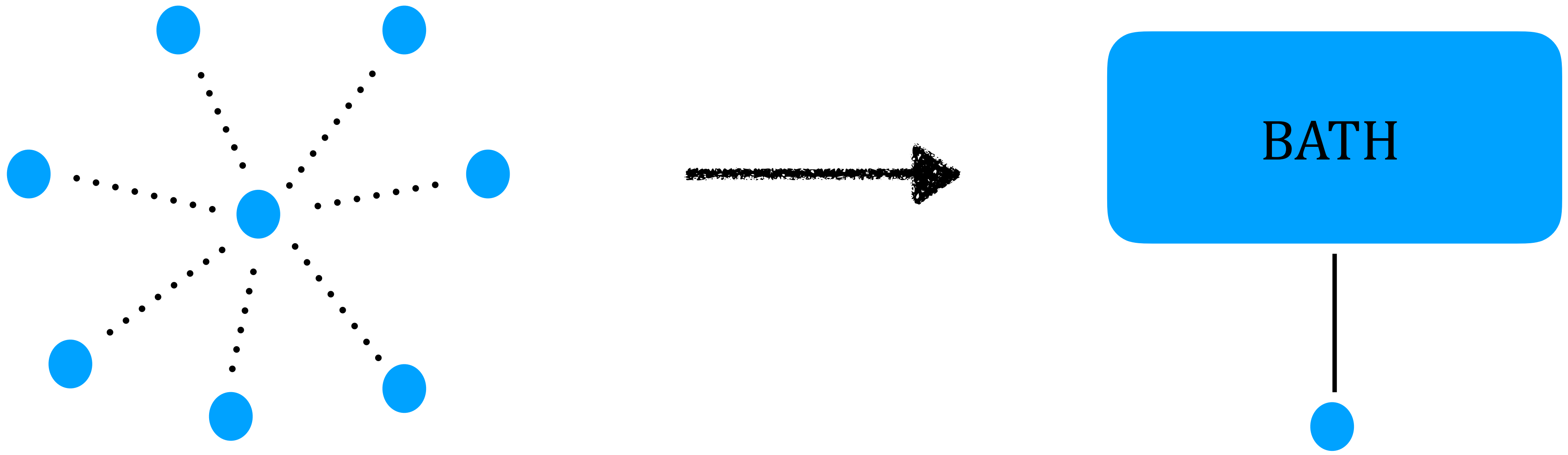
$-1 \leq \gamma \leq 1$

Bunin, Phys. Rev. E **95** 042414 (2017)
Barbier, Arnoldi, Bunin, Loreau PNAS **115** 2156 (2018)

Method: Dynamical Mean-Field Theory

Interactions with many species

Effective description



+

Self-Consistent Closure

Roy, GB, Bunin, Cammarota, J. Phys. A 2019

Questions

•What are the "phases" of ecosystems?

Emergent collective behaviours

→ This talk

•What are the factors determining-limiting diversity ?

Complexity versus stability in ecosystems

"Will a large complex ecosystem be stable?" R. May 1972

→ Guy's talk

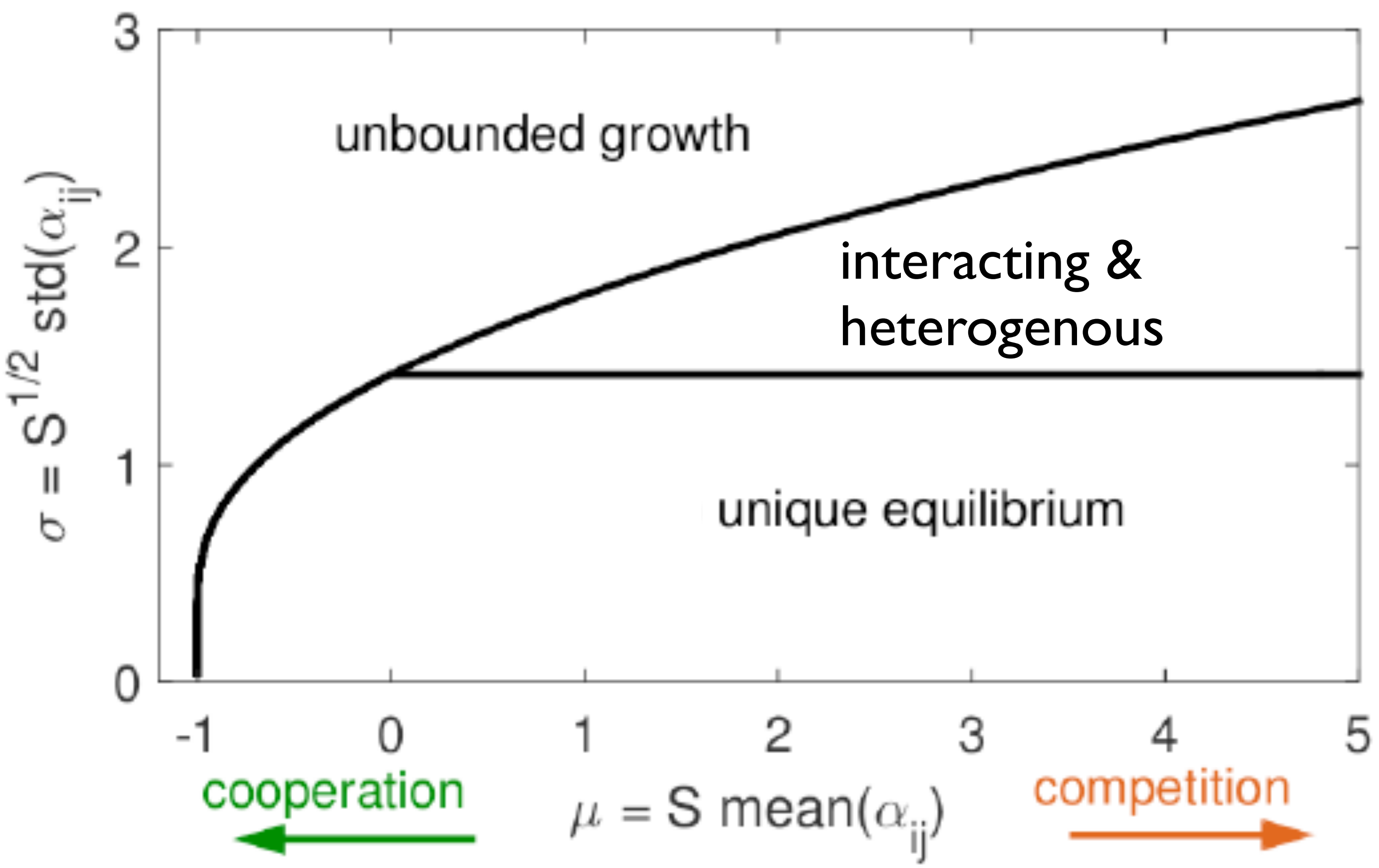
•Can endogenous fluctuations survive in a large interacting ecosystem?

Endogenous versus exogenous fluctuations

"Are ecological systems chaotic—and if not, why not?" Berryman, Millstein 1989

The Phase Diagram

For large S (>30) different “phases” emerge



The Phase Diagram

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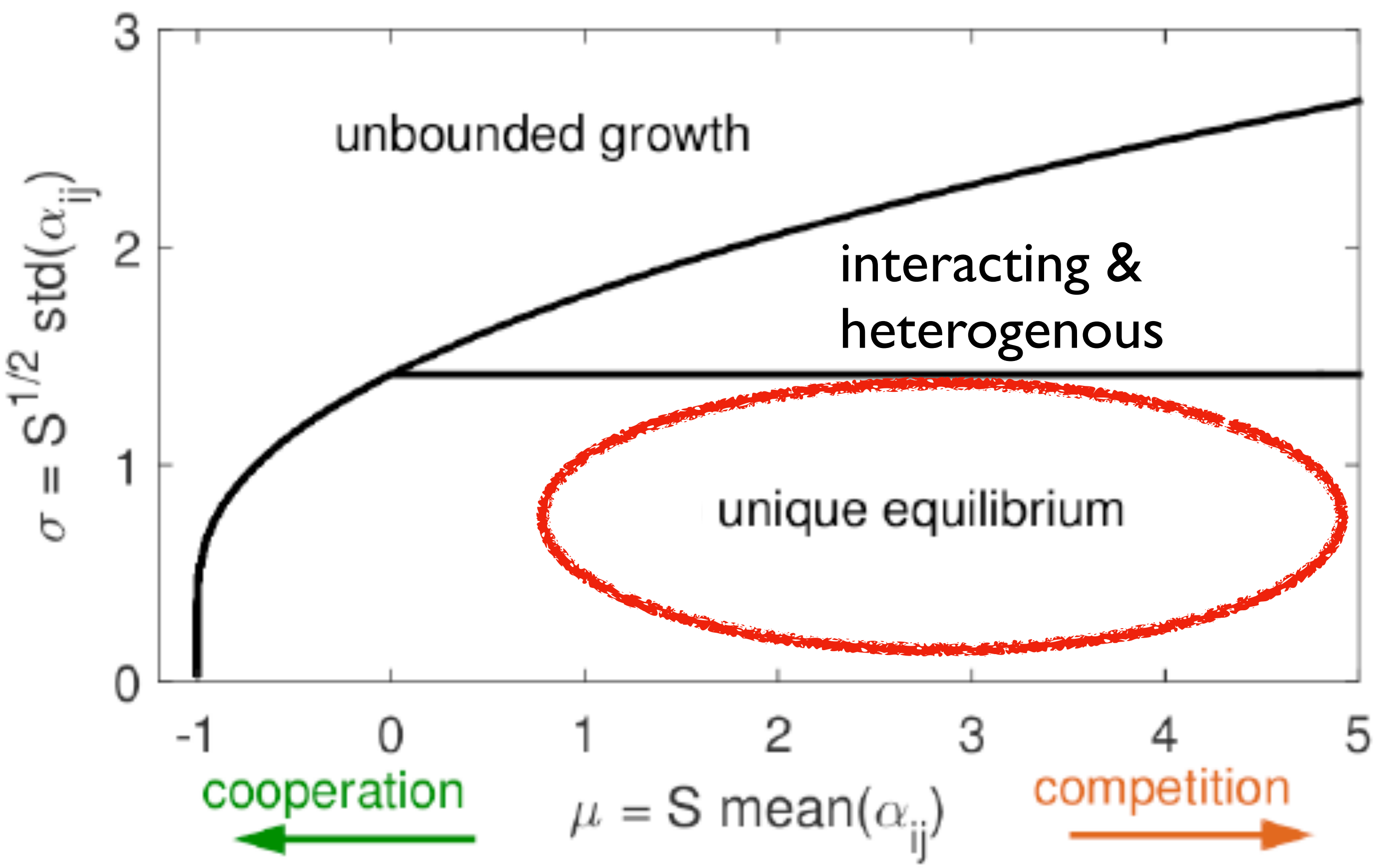
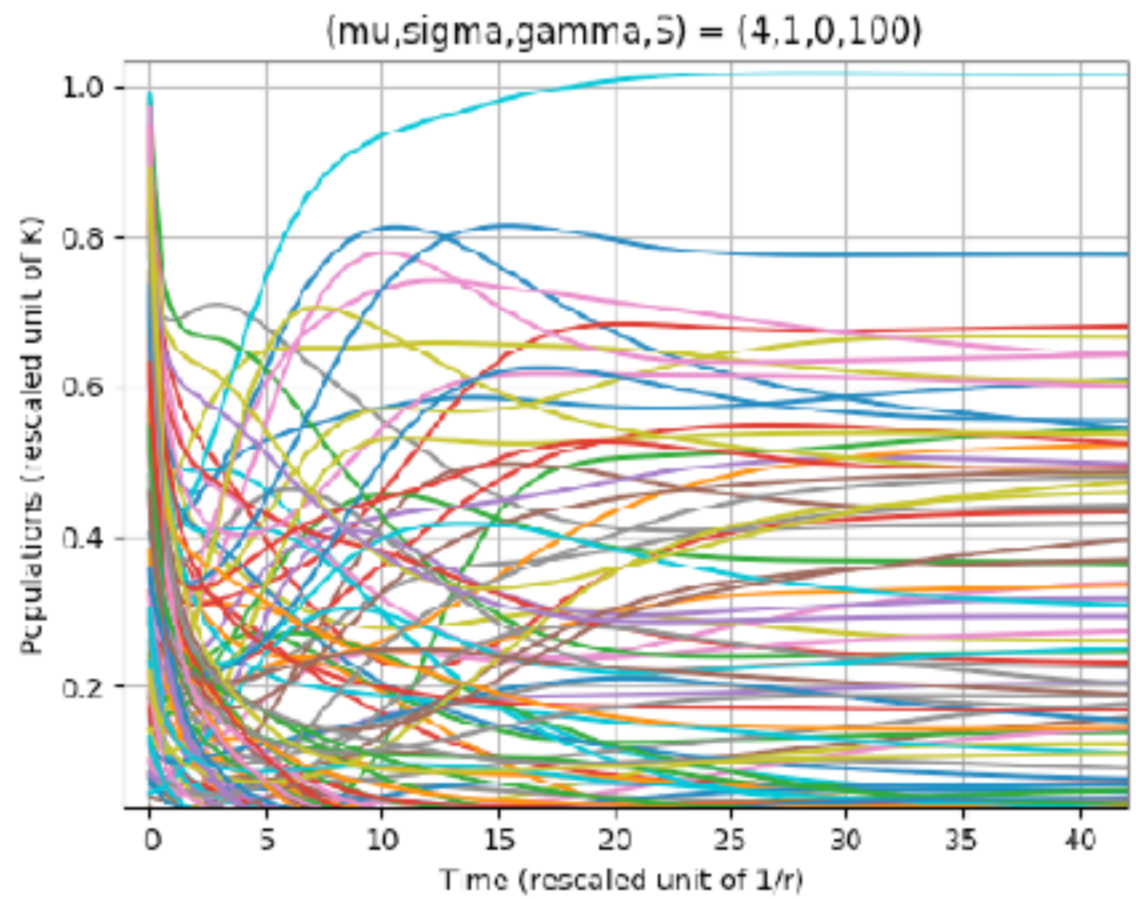
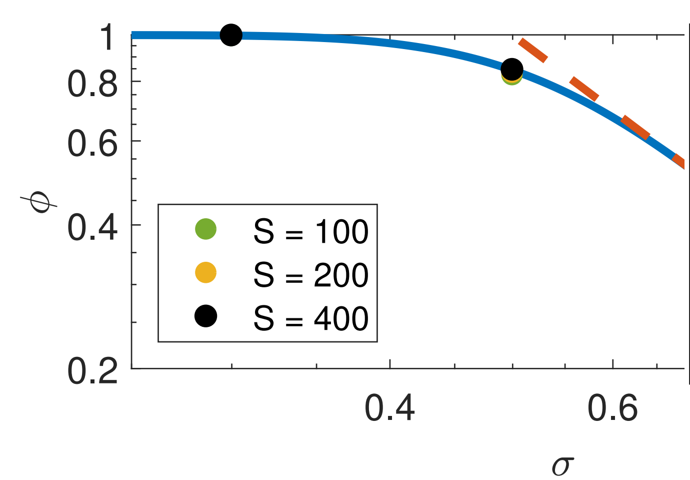


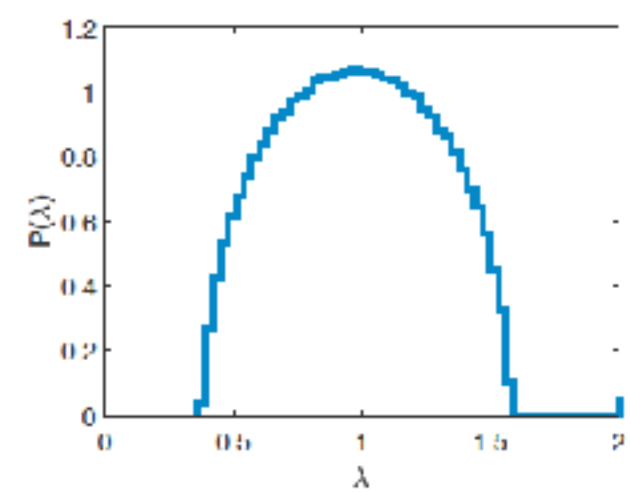
Unique Equilibrium Phase



- Diversity decreases with increasing heterogeneity in interactions
- Stability decreases



Density of eigenvalues of the stability matrix (symmetric case, GOE)

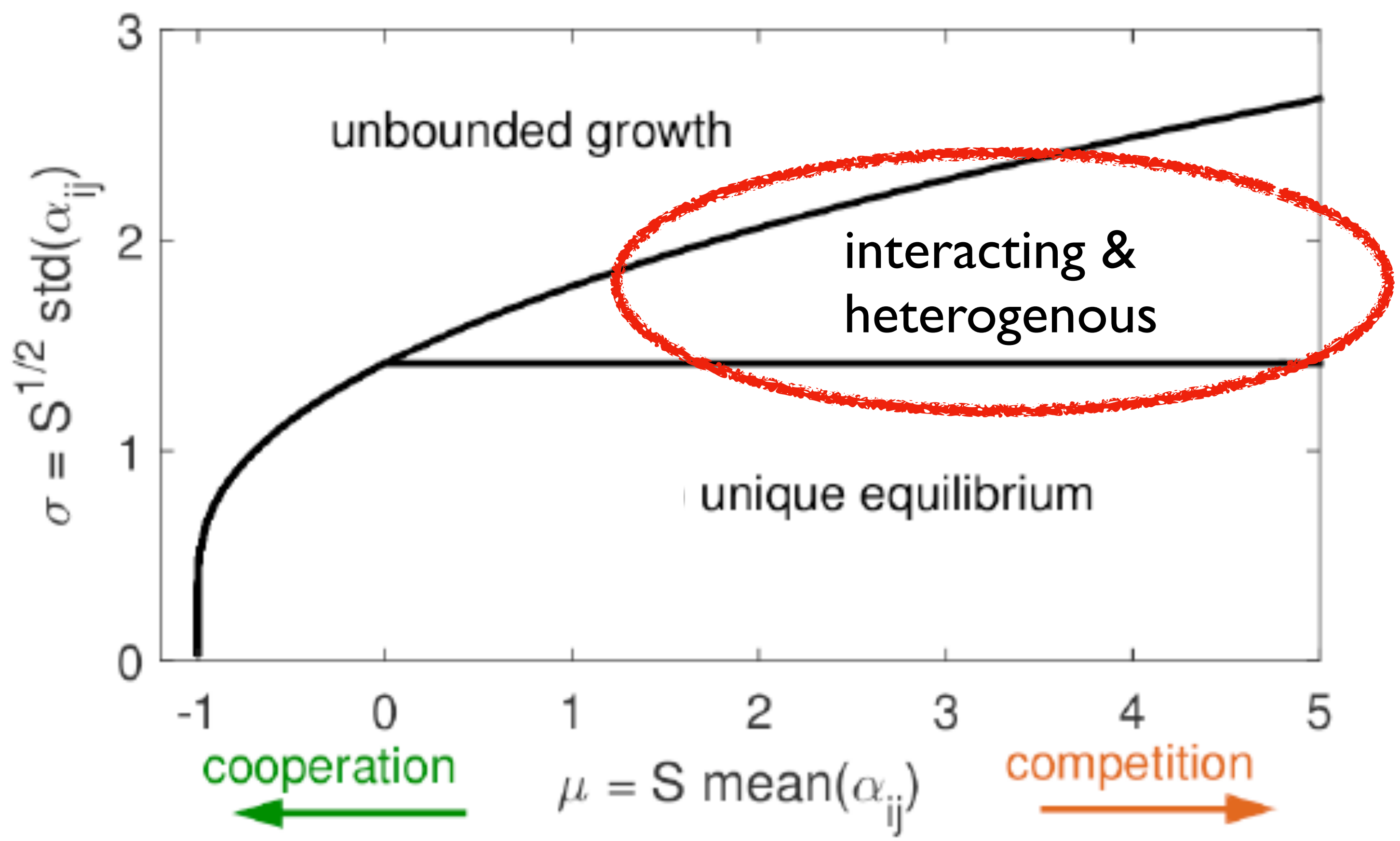


Bunin, Phys. Rev. E **95** 042414 (2017)

Interacting & Heterogeneous Regime

$$\sigma > \sigma_c$$

Beyond the stability limit of the one equilibrium phase (May's limit)

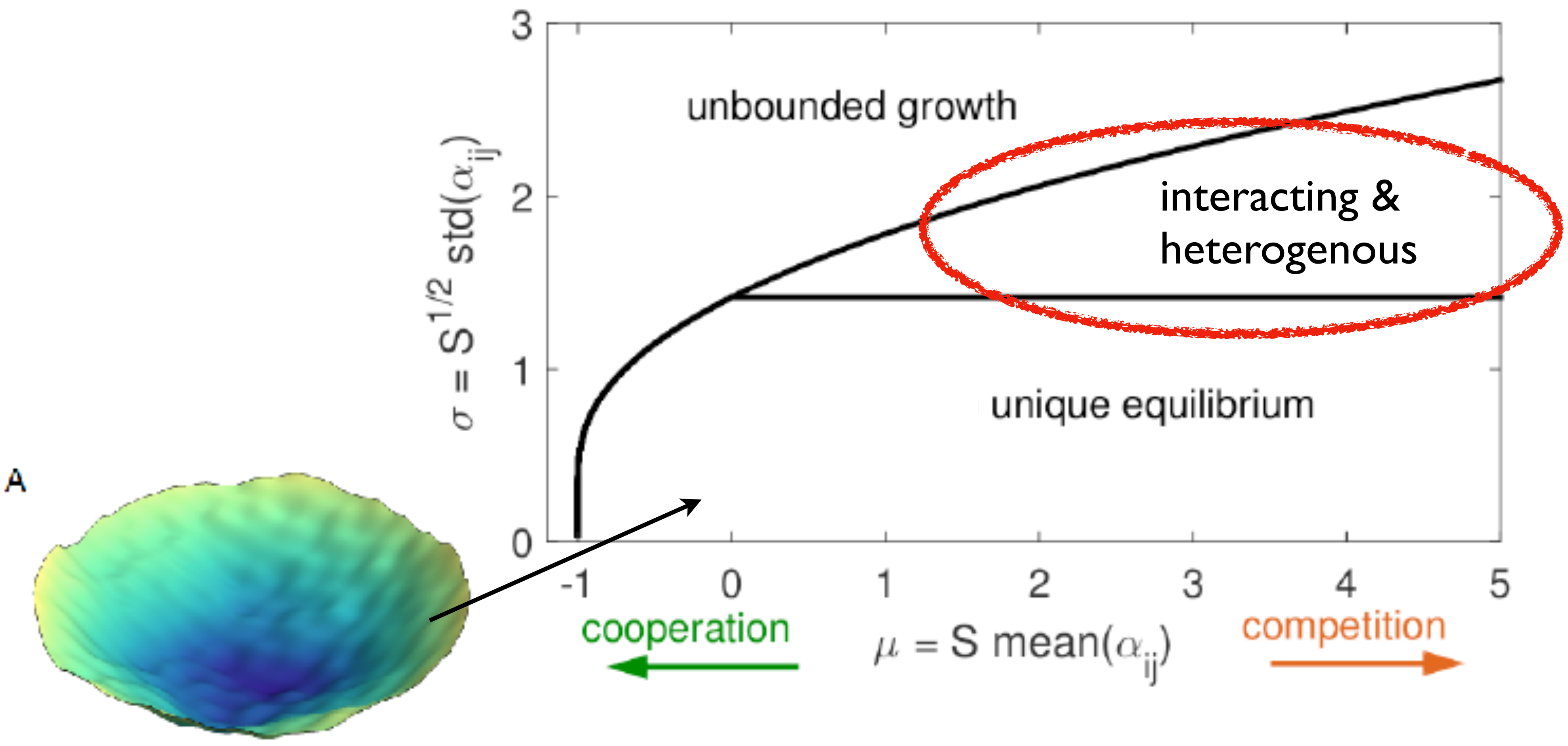


A particular case: symmetric interactions

$\gamma = 1$

$$\frac{dN_i}{dt} = -N_i \partial_{N_i} E(\{N_i\}) \qquad E = \sum_i \left(\frac{N_i^2}{2} - N_i \right) + \frac{1}{2} \sum_{i \neq j} \alpha_{ij} N_i N_j$$

Quench dynamics of a disordered system



A particular case: symmetric interactions

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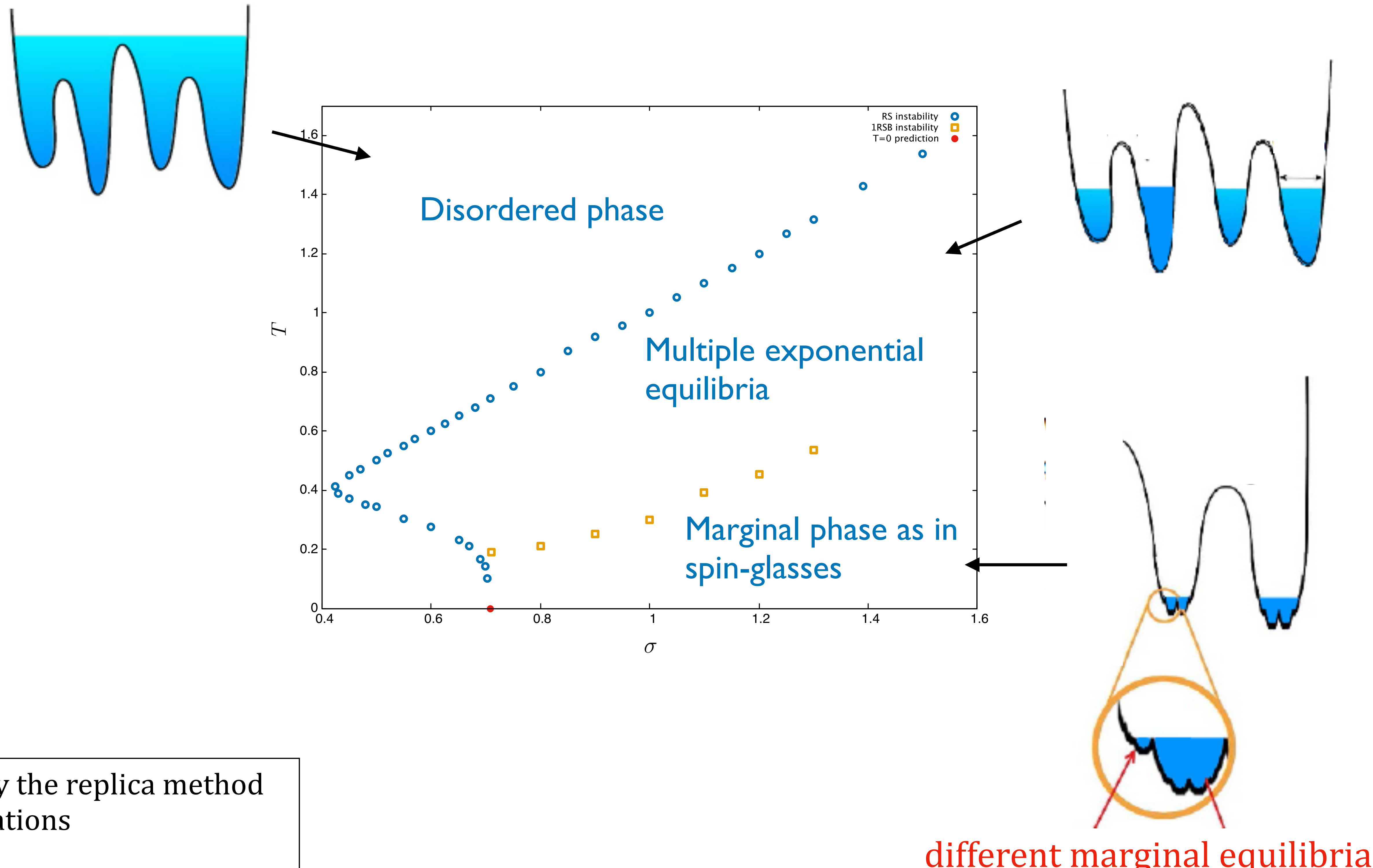


Analysis by the replica method and simulations

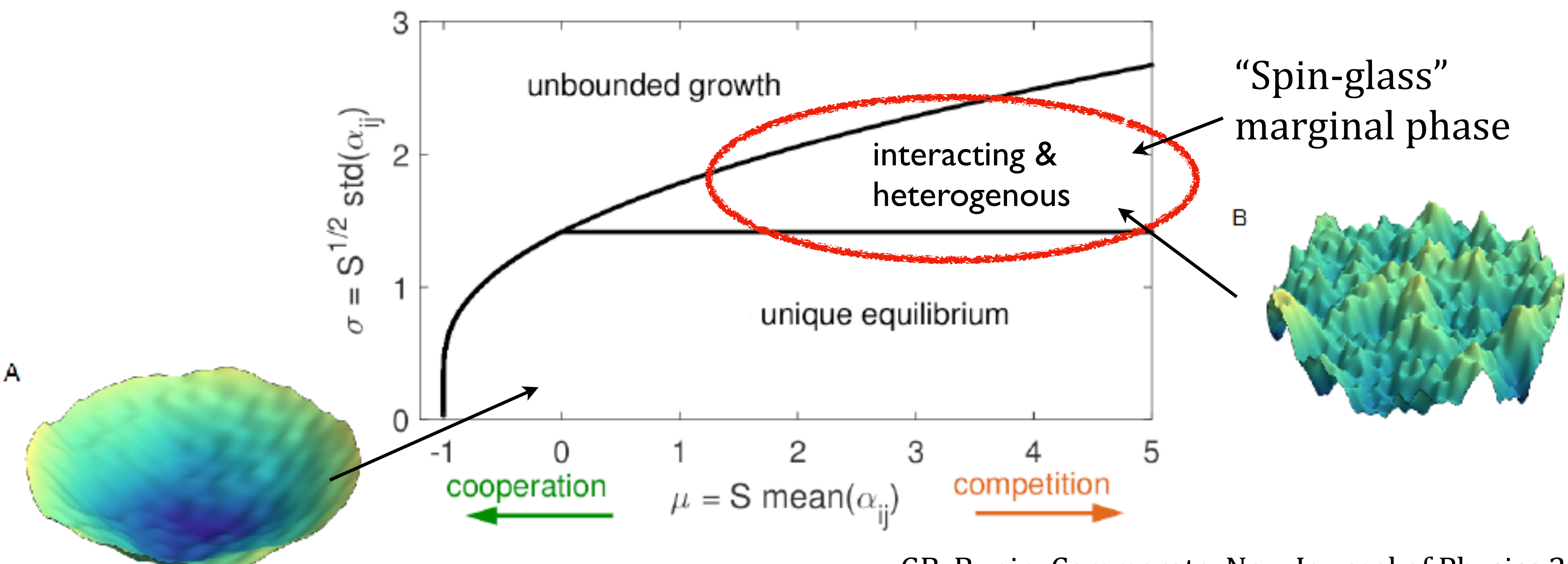
A. Altieri, F. Roy et al (to appear)

A particular case: symmetric interactions

$$\gamma = 1$$

$$\frac{dN_i}{dt} = -N_i \partial_{N_i} E(\{N_i\}) \qquad E = \sum_i \left(\frac{N_i^2}{2} - N_i \right) + \frac{1}{2} \sum_{i \neq j} \alpha_{ij} N_i N_j$$

aging dynamics and asymptotic marginal stability
as in mean-field spin-glasses



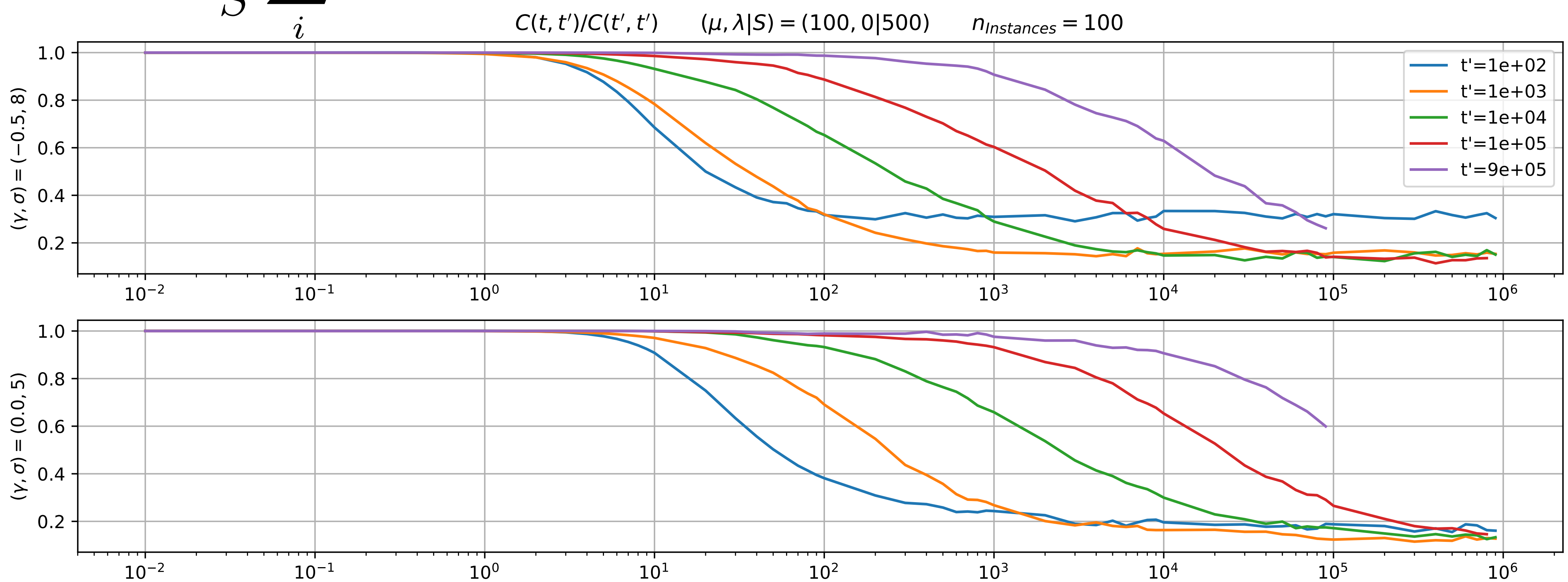
GB, Bunin, Cammarota, New Journal of Physics 2018
A. Altieri, F. Roy et al (to appear)

Without immigration

Aging Dynamics

$$C(t,t') = \frac{1}{S}\sum_i N_i(t)N_i(t')$$

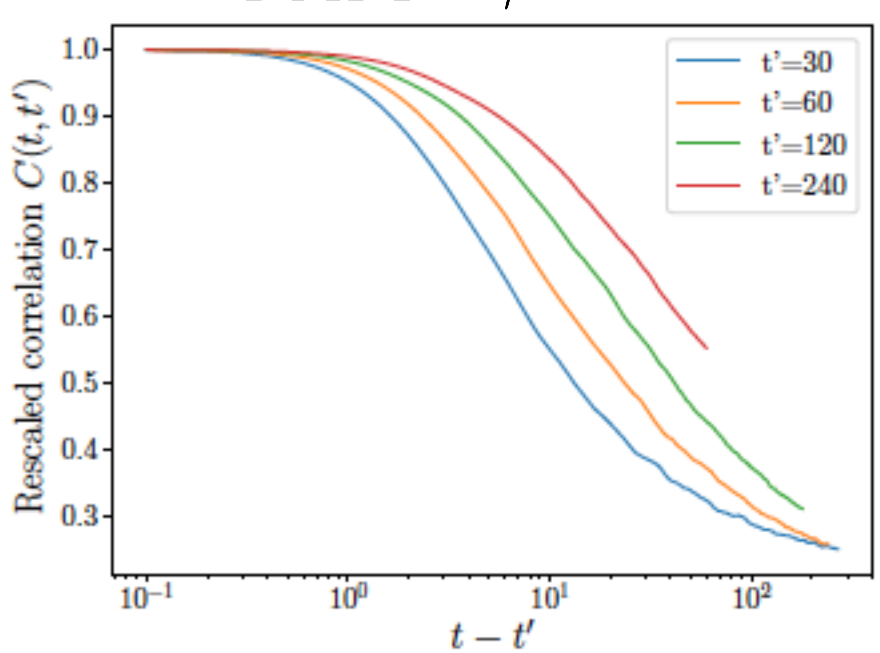
Numerical Simulations



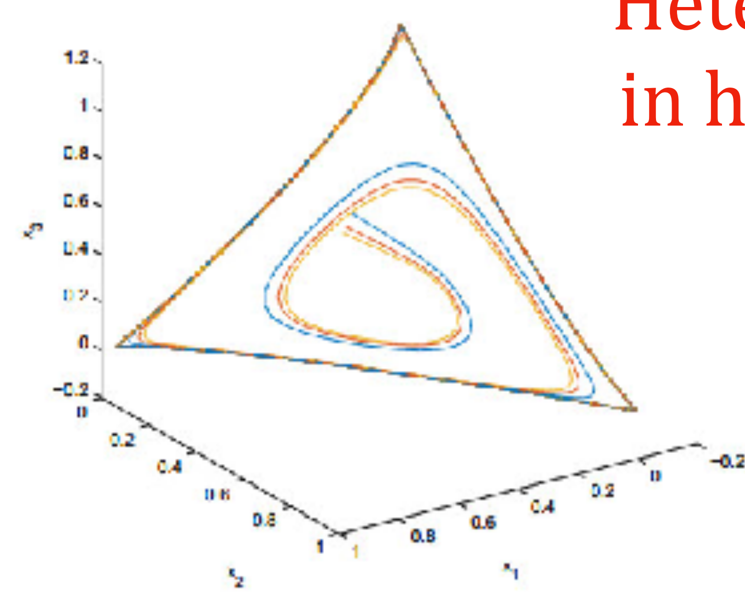
$\gamma = -0.5$

$\gamma = 0$

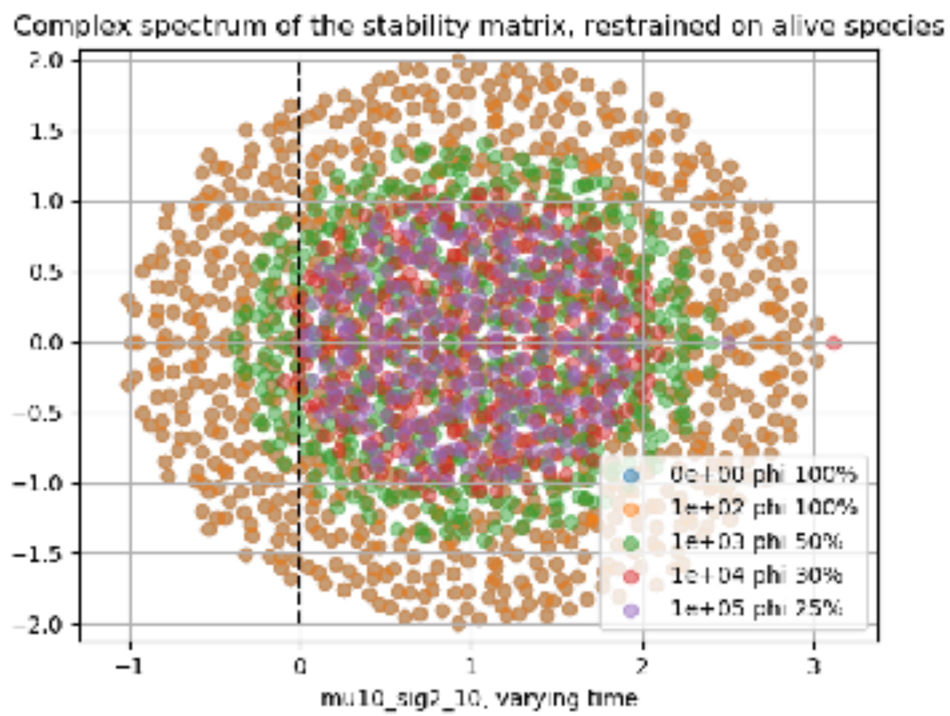
DMFT $\gamma = 0$



Heteroclinic chaos in high dimension



Critical Phase
Marginal stability & Diversity



- Marginal stability sets in dynamically

- Criticality at long times: large susceptibility to perturbations

- Diversity sets by marginal stability at the May bound

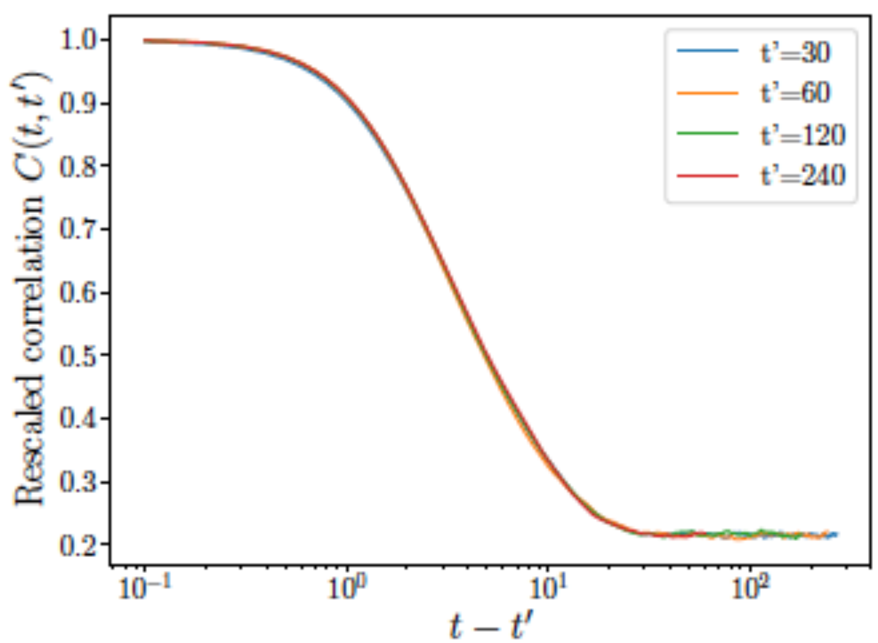
Solé, Alonso, McKane ‘02

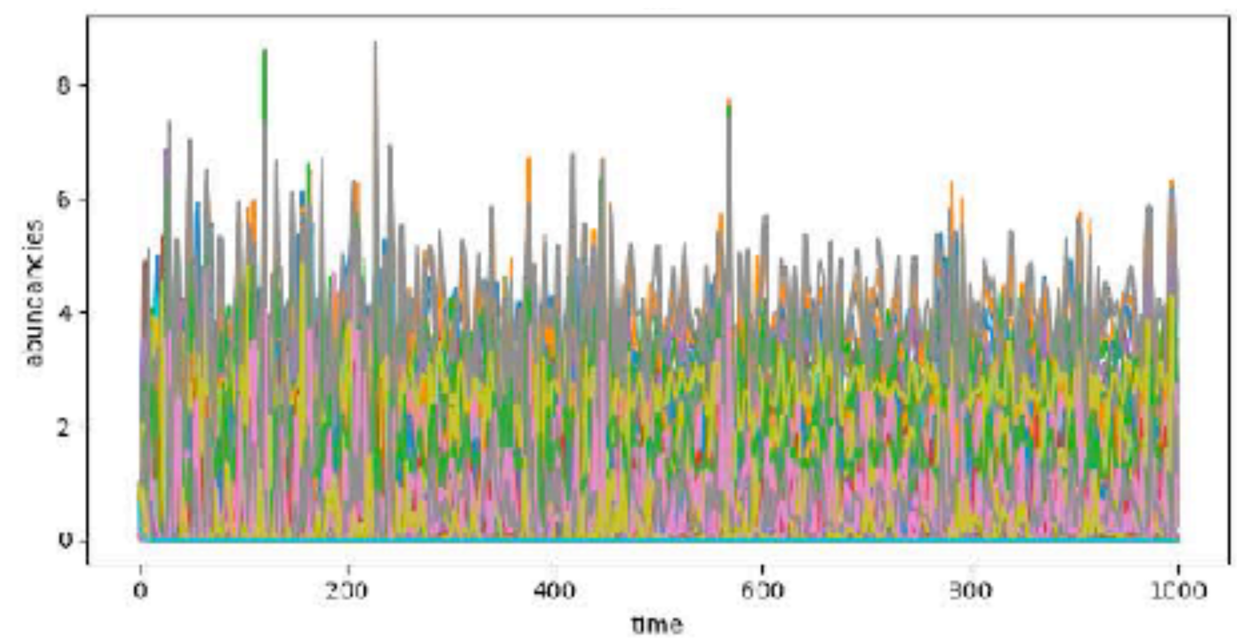
GB, Bunin, Cammarota, New Journal of Physics 2018

F. Roy et al to appear

Immigration is a singular perturbation -> chaos

$$\frac{dN_i}{dt} = N_i \left[1 - N_i - \sum_{j \neq i} \alpha_{ij} N_j \right] + \lambda$$





$\gamma = 0$

Endogenous fluctuations → Guy's talk

Roy, Barbier, Biroli, Bunin arXiv 1908.03348

Related works: Sompolinsky, Crisanti, Sommers '88 ; Kessler, Shnerb '15

Thank You!

Some references

- Generalized Lotka-Volterra Model & Random interactions

R. May, Nature **238** 413 (1972)
Barbier et al. PNAS **115** 2156 (2018)

- Phases of ecosystems, marginality and diversity

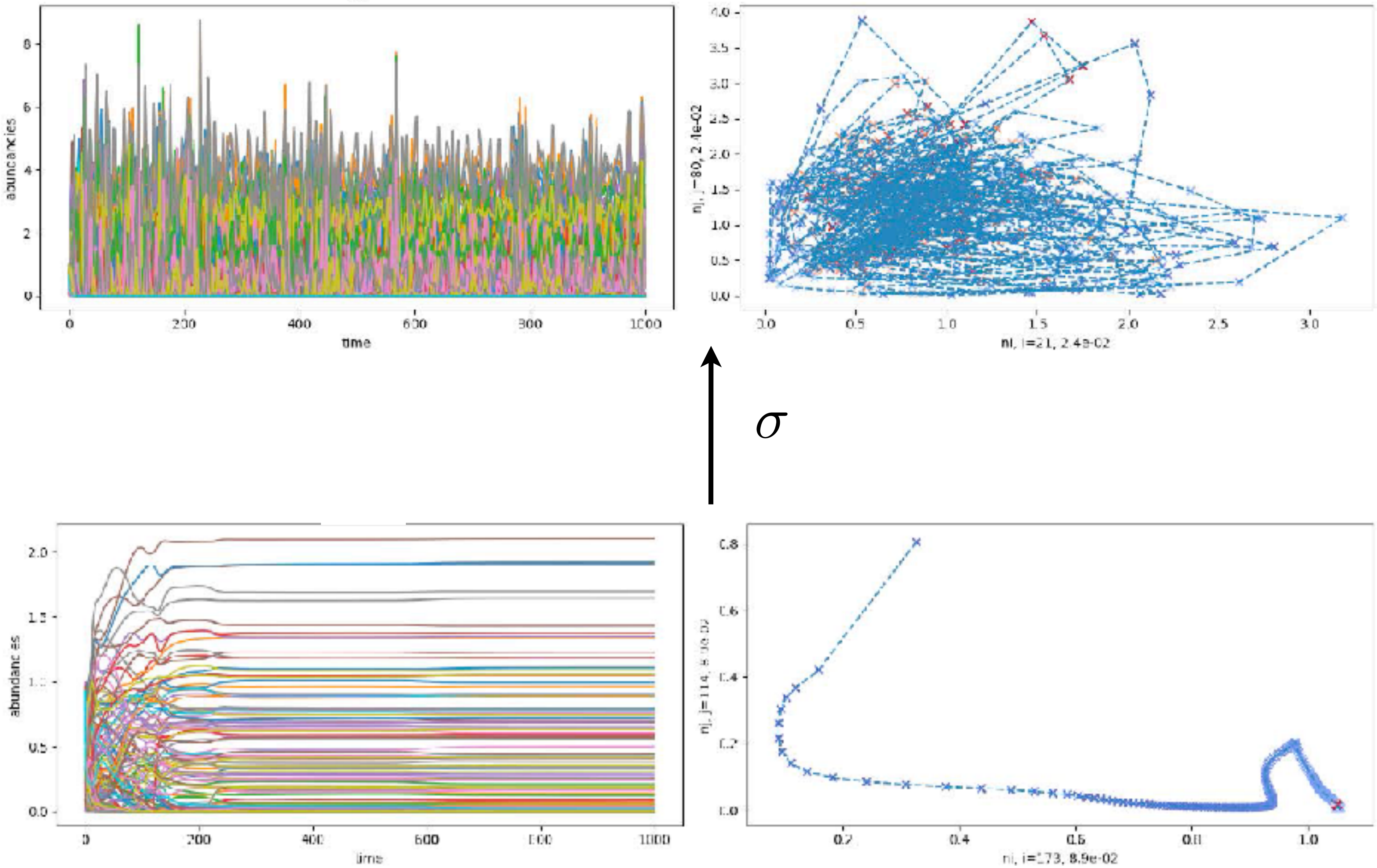
Kessler, Shnerb, Phys. Rev. E **91** 042705 (2015)
Bunin, Phys. Rev. E **95** 042414 (2017)
Solé, Alonso, McKane, Phil. Trans. Roy. Soc. B **357** 667 (2012)
Bunin, Biroli, Cammarota, New Journal of Physics **20** 2018

- Chaos & Endogenous fluctuations

Berryman, Millstein, Trends in Ecology & Evolution, **4** 26 (1989)
Nisbet et al, Trends in Ecology and Evolution, **4** 238 (1989)
Roy, Barbier, Biroli, Bunin arXiv 1908.03348
Pearce, Agarwala, Fisher, https://doi.org/10.1101/736215

Transition to Chaos

$\gamma < 1$



Method: Mean-Field Theory

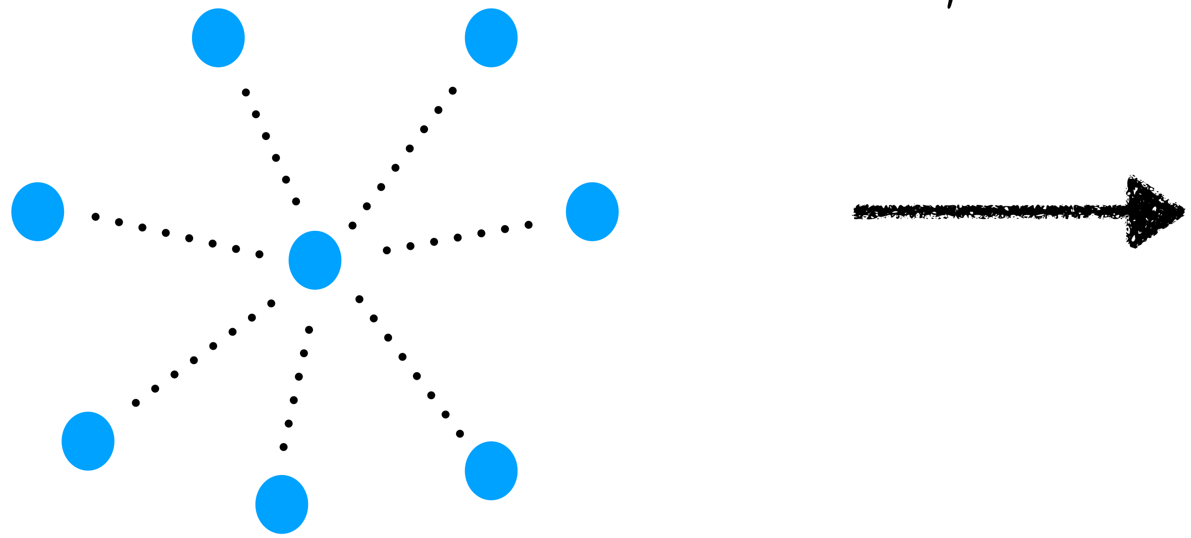
$$\frac{dN_i}{dt} = N_i \left[r_i(K_i - N_i) - \sum_{j,(j\neq i)} \alpha_{ij} N_j \right] + \sqrt{N_i}\eta_i(t) + \lambda_i$$

$$\frac{dN_i}{dt} = N_i \left[r_i(K_i - N_i) - \mu m(t) - \sigma\eta(t) + \gamma\sigma^2 \int_0^t \chi(t,s)N(s)ds + h(t) \right]$$

$\eta(t)$ Gaussian noise

$$\mu = \frac{1}{S}\sum_j \alpha_{ij} \qquad \sigma^2 = \sum_j (\alpha_{ij} - \mu/S)^2$$

γ correlation



noise + "friction"

Method: Mean-Field Theory

$$\frac{dN_i}{dt} = N_i \left[r_i(K_i - N_i) - \sum_{j,(j \neq i)} \alpha_{ij} N_j \right] + \sqrt{N_i}\eta_i(t) + \lambda_i$$

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$\eta(t)$ Gaussian noise $\qquad E[\eta(t)\eta(s)] = C(t,s)$

Self-consistent closure

$$C(t,s) = \frac{1}{S}\sum_j N_j(t)N_j(s)$$

$$m(t) = \frac{1}{S}\sum_j N_j(t) \qquad \chi(t,s) = \frac{1}{S}\sum_j \left.\frac{\delta N_j(t)}{\delta h_j(s)}\right|_{h_j=0}$$